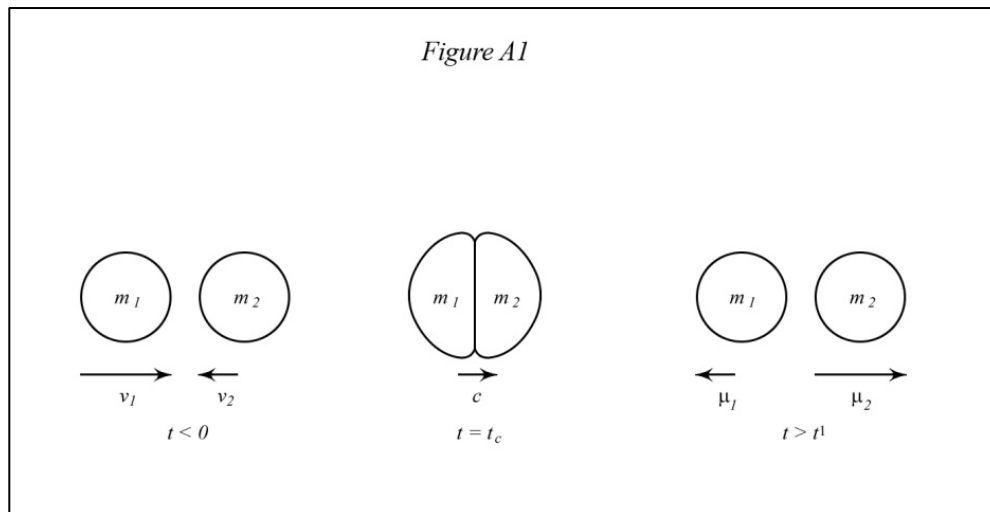


## Appendix

Taken from Freier's book

This discussion is taken from George D. Freier's book **University Physics: Experiment and Theory** published in 1965.<sup>1</sup> This physicist (Alfred Phillips, Jr.) was introduced to this book as a TA under Professor Herman Branson at Howard University. The content of this appendix is contained in many books, e.g., Landau and Lifshitz's **Mechanics**, Goldstein's **Classical Mechanics**, **The Feynman Lectures on Physics** (which this physicist has read in its entirety at least three times). Each of these expositions is based upon Newton's *Philosophiæ Naturalis Principia Mathematica* published in 1687. Freier's book abounds with physical experiments that students can do on the bench or in the mind. This physicist, as a kid, often rode his bike over to Chicago's *Museum of Science and Industry* where he played with an array of about a dozen ceiling-strung bowling-size balls such as those described in the last part of this appendix. The details of Freier's analysis are given for the readers of this paper and the high school and college students that NSBP (National Society of Black Physicists) reaches out to.

Masses,  $m_1$  and  $m_2$ , travel in a head-on manner with velocities,  $v_1$  and  $v_2$ , and collide at time zero. The collision lasts for duration  $t'$ . The maximum deformation occurs at  $t_c$  half way through  $t'$ . Figure A1 gives a schematic of the collision and Figure A2 gives an idealized representation of the forces as



**Figure A1** Mass  $m_1$  having initial velocity  $v_1$  moves towards mass  $m_2$  which has velocity  $v_2$ . At time  $t_c$  both masses have a common velocity  $c$ ; the deformations are maximal and we show them exaggerated. After the collision the masses move away with velocities  $u_1$  and  $u_2$ , respectively.

a function of time.  $F_1$  is the force on  $m_1$  caused by  $m_2$  from the *Action-Reaction Law* and

conversely for  $F_2$ . The velocities after collision are  $u_1$  and  $u_2$ . (We do not ascribe the *Action-Reaction Law* to Newton. It is likely of African in origin as is essential idea in universal gravitation, and as is monotheism. Newton (Bacon earlier), and others were aware of books and ideas originally cognized in Africa. The true history of these ideas, as well as that of the so-called *Pythagorean Theorem*, may one day be re-discovered. Isaac Newton would, nevertheless, get my vote for the best scientist in the last one thousand years although Albert Einstein is my favorite.)

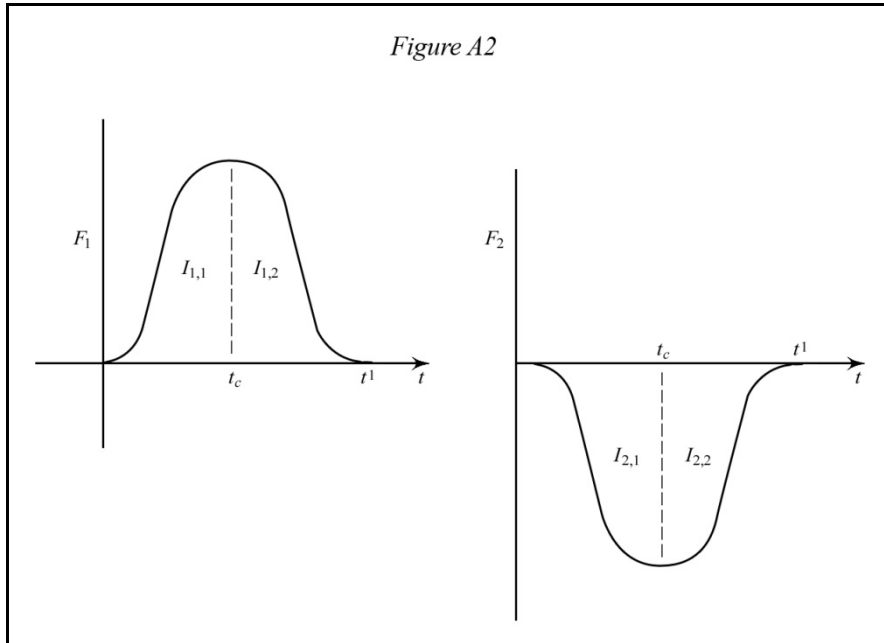


Figure A2 The graphs show how the forces  $F_1$  and  $F_2$  change from initial contact of  $m_1$  and  $m_2$  at  $t = 0$ , to the time of maximum deformation at time  $t_c$  to the time  $t'$  when the masses move apart.

The Action-Reaction law in equation form is

$$\mathbf{F}_1 = -\mathbf{F}_2.$$

The impulse integrals are

$$\int_0^{t'} \mathbf{F}_1 dt = - \int_0^{t'} \mathbf{F}_2 dt.$$

Using “Newton’s second law” for  $F_1$  and  $F_2$  . . .

$$\int_0^{t'} \mathbf{F}_1 dt = \int_{v_1}^{u_1} \frac{d(m_1 v)}{dt} dt = m_1 u_1 - m_1 v_1,$$

$$-\int_0^c F_2 dt = -\int_{v_2}^{u_2} \frac{d(m_2 v)}{dt} dt = -(m_2 u_2 - m_2 v_2),$$

$$m_1 u_1 - m_1 v_1 = -(m_2 u_2 - m_2 v_2).$$

Rearranging terms we obtain the law of *Conservation of Linear Momentum* . . .

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

We can get at Newton's *coefficient of restitution*,  $\epsilon$  (which is one for elastic collisions) by considering the instant of time  $t_c$  when the two masses have maximum deformation. We break the impulse integral into two parts (0 to  $t_c$  and  $t_c$  to  $t$ ) for  $m_1$  and  $m_2$ .

$$I_{1,1} = \int_0^{t_c} F_1 dt = \int_{v_1}^c \frac{d(m_1 v)}{dt} dt = m_1 c - m_1 v_1$$

$$I_{1,2} = \int_{t_c}^t F_1 dt = \int_c^{u_1} \frac{d(m_1 v)}{dt} dt = m_1 u_1 - m_1 c$$

$$-I_{2,1} = -\int_0^{t_c} F_2 dt = -\int_{v_2}^c \frac{d(m_2 v)}{dt} dt = -(m_2 c - m_2 v_2)$$

$$-I_{2,2} = -\int_{t_c}^t F_2 dt = -\int_c^{u_2} \frac{d(m_2 v)}{dt} dt = -(m_2 u_2 - m_2 c)$$

As this collision take place along a straight line, we consider scalar equations henceforth.

$$s_1 = \frac{I_{1,2}}{I_{1,1}} = \frac{u_1 - c}{c - v_1}$$

$$s_2 = \frac{I_{2,2}}{I_{2,1}} = \frac{u_2 - c}{c - v_2}$$

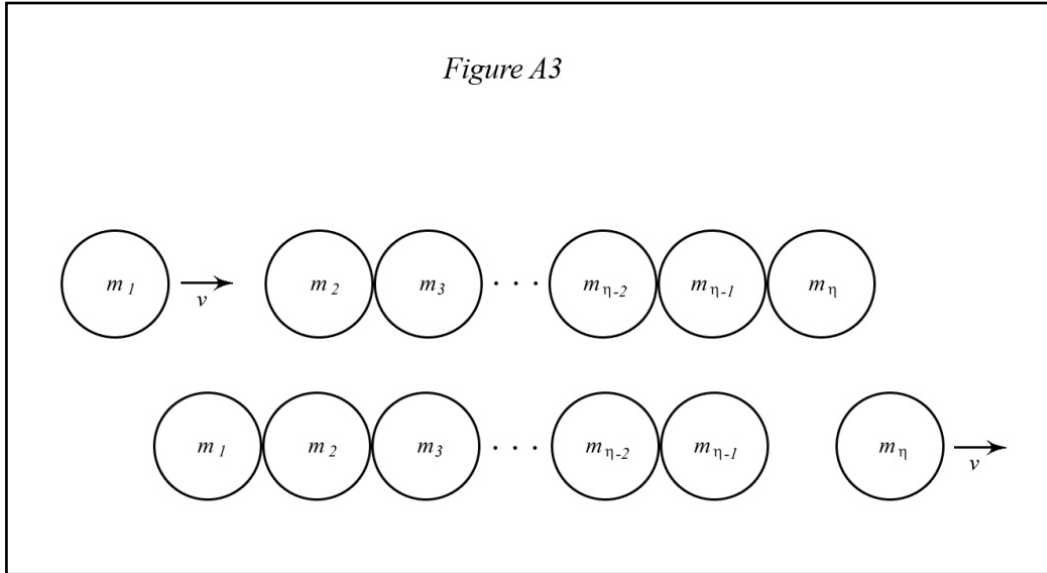
It also follows that the *coefficients of restitution* are equal from the *Action-Reaction Law*,

$$s_1 = s_2 = s.$$

We can find eliminate  $c$  between the two coefficients of restitution equations and find  $\epsilon$ .

$$s = -\frac{u_2 - u_1}{v_2 - v_1}.$$

Consider an array of identical balls each having mass,  $m$ , and are separated by an infinitesimally small distance. Let another identical ball strike the array from the left in an elastic manner along the straight line of their centers (see Figure A3). We will label the striking ball as  $m_1$  and the ball that is struck (the first ball in the array) as  $m_2$ . As the separation is infinitesimal, we can consider the collision between ball  $m_1$  and ball  $m_2$ . Since  $v_2$  is zero, the



**Figure A3** The ball labeled  $m_1$  travelling with velocity  $\mathbf{v}$  collides with an array of  $\eta-1$  identical balls that are arranged in a collinear manner. The ball labeled  $m_{\eta}$  is ejected with velocity  $\mathbf{v}$ . It is as if a virtual ball travelled through the apparently motionless array of the other  $\eta-2$  identical balls.

equation for the conservation of linear momentum is

$$m_1 v_1 + m_2(0) = m_1 u_1 + m_2 u_2,$$

and the equation for the *coefficient of restitution* is

$$e = -\frac{u_2 - u_1}{0 - v_1} = 1.$$

As the masses are identical, i.e.,  $m_1 = m_2 = m$ , we immediately find that

$$u_2 = 0, \text{ and}$$

$$u_1 = v_1.$$

In words, as we started with ball  $m_1$  moving with velocity  $v_1$ , and ball  $m_2$  stationary, after the collision,  $m_1$  is stationary and ball  $m_2$  moves with velocity  $v_1$ . As there is an infinitesimal separation between the balls the collisions are independent. Yet immediately after the first

collision there is a collision between the second and third ball. We assume that all of the collisions are elastic. We do not see the impact of this collision in terms of motion of the second and third balls. All we see is the last ball (the right most) moving out with velocity  $v_I$ . If we had injected several (say sixteen balls or any number you would pick) into the array of an unspecified number of initially still balls, we would see the same number (sixteen for our pick) of injected balls coming out of the far right side. One must think of these as collisions which occur in pairs or the results are more challenging to interpret. One thinking in the simpler independent collision manner would also be correct in this case. It would also be correct to think of the velocity (whence momentum, and kinetic energy) as being passed from ball-to-ball in this way, although some of the balls, when in close contact, do not appear to move.

To recapitulate and expand, one ball strikes an array of identical co-linear balls from the left and passes as a virtual ball along the array of balls with the right-most ball ejected with the same velocity as the original striking ball. *The energy and momentum of the virtual ball are passed through the array in the form of compressions and expansions. These compressions and expansions are disturbances and restorations of electric charges from and back to their space charge neutral positions.* The changes in the electrical charge distribution are governed by *Uncertainty* and *Exclusion Principles*. (This is similar to Feynman's reasoning on why our feet don't go through the floor as we walk.<sup>2</sup>)

If, in the original array, the last ball is fixed, the striking ball(s) will quickly emerge from the array with a velocity of  $-v_I$ . This would be equivalent a ball striking a massive plate as Freier shows in Figure 4-18. In his discussion he has a rare "typo." His expression should be  $\epsilon = -(0 - u_I)/(0 - v_I)$ . Of course, Freier's book is out of print and your library likely does not have such an inconsequential book from the sixties. That's another reason for this appendix. (As of April 11, 2008 *Amazon* had four copies, ranging in price from \$5 to more than \$111.)

<sup>1</sup>Freier, G. D. (1965). **University Physics: Experiment and Theory**. Appleton-Century-Crofts, New York. pp 42-45.

<sup>2</sup>Feynman, R. P., Leighton, R. B., Sands, M. **The Feynman Lectures on Physics: Quantum Mechanics, III**. Addison-Wesley Publishing Company, Reading, MA, etc. pp 2-6 and 2-7.